RELATIVISTIC FADDEEV APPROACH TO THE NJL MODEL AT FINITE DENSITY

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We study the nucleon solution of the relativistic Faddeev equation as a function of density in the framework of a generalized Nambu–Jona-Lasinio model. We truncate the interacting two-body channels to the scalar diquark channel, the coupling constant of which is treated as a parameter. A 3-momentum cut-off is used to regularize the model. The Faddeev equation is solved numerically using the methods developed by Tjon and others. At zero density the nucleon is bound only for unrealistically large values of the scalar coupling and this binding energy decreases quickly with increasing density.

1 Introduction

In contrast to high-temperature zero-density QCD, rather little is known about high-density zero-temperature QCD. Due to technical difficulties (the fermionic determinant becoming complex at finite chemical potential), lattice calculations are not able to provide unambiguous results. However, models of QCD seem to indicate a rich phase structure in high density quark matter. In particular, much attention has recently been devoted to so-called colour superconductivity [1,2]: at high density, an arbitrarily weak attraction between quarks makes the quark Fermi sea unstable with respect to diquark formation and induces Cooper pairing of the quarks (diquark condensation). However, the groups who have studied colour superconductivity focused only on instabilities of the Fermi sea with respect to diquarks and have not considered possible 3-quark clustering. To address this question would necessitate in principle a generalisation of the BCS treatment. As a first step we can look for instabilities of the quark Fermi sea with respect to 3-quark clustering by studying the evolution of the nucleon binding energy with density. A bound nucleon at finite density would be a signal of instability. In this study, we will use the Nambu–Jona-Lasinio (NJL) model and solve the relativistic Faddeev equation for the nucleon as a function of density.

2 The model

The NJL model provides a simple implementation of dynamically broken chiral symmetry. It has been successful in the description of mesonic properties at low energy and several groups [3,4,5] have used it to study baryons at zero density (for a review of the NJL model, see for example [6]). Several versions of the NJL lagrangian are available. Whatever the version we choose, a Fierz transformation allows us to order the terms according to their symmetries in the $q\bar{q}$ channel; here we are only interested in the scalar and pseudoscalar terms:

$$\mathcal{L}_\pi = \frac{1}{2} g_\pi [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau\psi)^2] \quad . \tag{1}$$

To study the baryons, another Fierz transformation has to be performed in the qq channel. In this work we shall keep only the scalar diquark channel:

$$\mathcal{L}_s = g_s(\bar{\psi}(\gamma_5 C)\tau_2\beta^A\bar{\psi}^T)(\psi^T(C^{-1}\gamma_5)\tau_2\beta^A\psi) \quad , \tag{2}$$

where $\beta^A = \sqrt{3/2}\,\lambda^A$ for $A = 2, 5, 7$ projects on the colour $\bar{3}$ channel and $C = i\gamma_2\gamma_0$ is the charge conjugation matrix.

The ratio g_s/g_π depends on the version of the NJL lagrangian used. In the following, we will not choose a particular version of the model but rather leave the ratio g_s/g_π as a free parameter. We regularize the model with a 3-momentum cut-off Λ. We have two parameters g_π and Λ, which are fitted to the values of the pion decay constant $f_\pi = 93$ MeV and the constituent quark mass $M = 400$ MeV. This gives us $g_\pi = 7.01$ GeV^{-2} and $\Lambda = 0.593$ GeV.

The effect of density is introduced by imposing the quark 3-momentum to be larger than the Fermi momentum k_F. Solving the gap equation as a function of k_F gives us the usual dependence of the quark constituent mass on density. Chiral restoration occurs at $k_F/\Lambda = 0.58$, which corresponds to about 2.1 times the nuclear matter density

3 Diquark at finite density

As a first step to the resolution of the Faddeev equation, the scalar diquark mass has to be calculated as a function of density. The Bethe-Salpeter equation for the 2-body T-matrix is solved in the scalar qq channel using the ladder approximation (the explicit solution is given in[3,4]) and the pole of the T-matrix gives the mass of the bound diquark. Note that the denominator of the scalar qq T-matrix is formally identical to that of the pionic T-matrix, except for the replacement of g_s by g_π. That means that the pion and the scalar diquark are degenerate (and of zero mass) for a ratio $g_s/g_\pi = 1$. Fig. 1 gives the binding energy of the scalar diquark, $B_{diq} = 2\sqrt{k_F^2 + M^2} - E_{diq}$ as a function of the
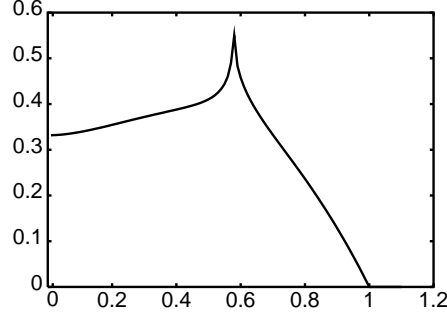
Figure 1: Diquark binding energy (in GeV) as a function of k_F/Λ

dimensionless variable k_F/Λ for a value of the scalar coupling $g_s/g_\pi = 0.83$. One can see that the scalar diquark is bound for all values of the density, the binding energy increasing with density to reach more than half a GeV. The sharp peak, which occurs at chiral symmetry restoration, is a consequence of the choice of a large scalar coupling, close to the value $g_s/g_\pi = 1$, for which the scalar diquark and the pion are degenerate.

4 The relativistic Faddeev equation

Because of the separability of the NJL interaction, the 3-body relativistic Faddeev equation in the ladder approximation can be reduced to an effective 2-body Bethe-Salpeter equation describing the interaction between a quark and a diquark. Details concerning the derivation of this equation can be found in the previous studies performed at zero density [3,4]. Explicitly, it is written:

$$\Psi(P,q) = \frac{i}{4\pi^4} \int d^4q' R(2P/3 + q')V(q,q';P)\Psi(P,q') \quad , \qquad (3)$$

where $R(2P/3 + q')$ is the two-body T-matrix for the scalar diquark and $V(q,q';P)$ involves the product of the propagators of the spectator and exchanged quarks:

$$V(q,q';P) = \frac{(\gamma p_2' + m)(\gamma p_1' + m)}{(p_1'^2 - m^2)(p_2'^2 - m^2)} \quad . \qquad (4)$$

Here p_i' (i=1,2,3) are the momenta of each valence quark, $P = p_1 + p_2 + p_3$ is the total momentum of the nucleon, and q, q' are the Jacobi variables defined

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by $p_3 \equiv P/3 - q$; $p'_1 \equiv P/3 - q'$. We now look for the nucleon solution of (3):

$$\Psi = \begin{pmatrix} \Phi_1(q_0, q) \\ \vec{\sigma}.\vec{q}\,\Phi_2(q_0, q) \end{pmatrix}. \tag{5}$$

With this form for Ψ, Eq. (3) becomes a set of two coupled integral equations. Following Huang and Tjon[4], we then perform a Wick rotation on the q_0 and q'_0 variables. This leads to two coupled complex integral equations which are solved iteratively[4,7]. The initial guess for each of the wave functions $\Phi_1(q_0, q)$ and $\Phi_2(q_0, q)$ consists of a Gaussian for the real part and a derivative of a Gaussian for the imaginary part. The number of iterations needed to reach convergence of the solutions is about four.

The above discussion remains valid at finite density. Again, we incorporate the effects of density by restricting the 3-momentum of each valence quark to values larger than the Fermi momentum k_F, i.e.:

$$k_F \leq |\vec{p_i}'| \leq \Lambda \quad (i = 1, 2, 3). \tag{6}$$

This condition translates into Eq. (3) as a complicated cut-off on the 3-momentum integration variable. Apart from this restriction, the method of solving the Faddeev equation is the same as at zero density.

5 Results and conclusions

At zero density we found that the nucleon is bound only if the scalar coupling is strong enough, i.e. $g_s/g_\pi \geq 0.8$, which means that the nucleon is not bound in either the "standard" NJL model ($g_s/g_\pi = 2/13$) or the colour-current interaction version ($g_s/g_\pi = 1/2$).

At finite density, we solve the Faddeev equation for the energy E_{nuc} of the nucleon as a function of the Fermi momentum. Results are shown in Fig. 2 for $g_s/g_\pi = 0.83$. The binding energy of the nucleon, $B_{nuc} = E_{diq} + \sqrt{k_F^2 + M^2} - E_{nuc}$, is depicted, again as a function of the dimensionless variable k_F/Λ. The binding energy of the nucleon is relative to the quark-diquark threshold: as there is no confinement in the NJL model, nothing can prevent the existence of a free diquark. In contrast to the diquark (shown in Fig. 1 for the same value of g_s/g_π), which is bound over the whole range of densities, the binding energy of the nucleon decreases quickly with density and the binding disappears well before nuclear matter density (which corresponds to $k_F/\Lambda \simeq 0.45$).

These results imply that, in the region characteristic of colour superconductivity (beyond chiral symmetry restoration), the quark Fermi sea is unstable only with respect to formation of diquarks and not of 3-quark clusters. However, we have to emphasize that we included only the scalar part of the
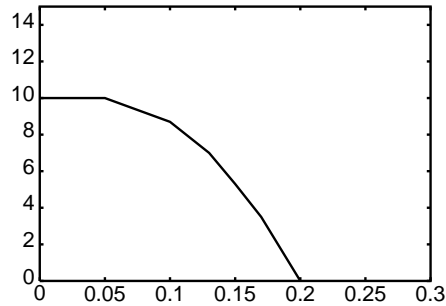
Figure 2: Nucleon binding energy (in MeV) as a function of k_F/Λ

qq interaction; at zero density, several authors [3,8] have shown that the axial-vector qq interaction gives an important contribution to the nucleon binding energy (of the order 100 MeV), while the axial-vector diquark is not bound for reasonable values of the axial-vector coupling. Incorporating the axial-vector qq interaction is necessary to obtain a more realistic picture of the evolution of the nucleon binding energy with density and could significantly modify the present results.

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